Contributions to the Founding of the Theory of Transfinite Numbers. By GEORG CANTOR. Translated by P. E. B. Jourdain. Open Court Company. Pp. ix, 208.

Mr. Jourdain and his publishers are deserving of the heartiest thanks from English philosophers and mathematicians for this excellent translation of Cantor's epoch-making articles in the *Mathematische Annalen* between 1895 and 1897. The book appears at a singularly appropriate moment for the refutation of that particular kind of fool who is fond of telling us that the German intellect has made no specially important contributions to knowledge. Perhaps nothing can be much more important to philosophy and mathematics than clear ideas about number, continuity, and infinity; and from the seventeenth century till quite recent years practically all the great contributors—Leibniz, Bolzano, Weierstrass, Dedekind, Schröder, Frege, and Cantor—have been citizens of the Central Powers. This of course is said without prejudice to the extremely valuable work that has been done in the last twenty years in Italy, England, and America; but there is no pressing danger of the work of allies and neutrals being slighted.

About two-fifths of the present volume is taken up with an introduction by Mr. Jourdain, in which he traces the connexion of Cantor's latest view with his earlier work and with that of his predecessors. And at the end he has added a few notes on the development of the subject since Cantor wrote these articles. The main developments have been (*a*) the purely logical definition of number which we owe to Frege and Russell; (*b*) the Schröder-Bernstein theorem; (*c*) Russell's and Whitehead's definitions of arithmetical operations (with a special view to dealing with the products and sums of infinite numbers of factors); (*d*) the rendering explicit of the Multiplicative Axiom; (*e*) the discovery of difficulties such as Burali-Forti's contradiction, and their solution by the Theory of Logical Types.

In the introduction Mr. Jourdain starts with Fourier, and the problems which his development of functions in trigonometrical series raised. The treatment of these problems by Dirichlet led to the theory of functions of a real variable. Cauchy and Weierstrass, on the other hand, developed the theory of functions of a complex variable. Riemann, influenced by both Dirichlet and Cauchy, greatly developed this theory, and Hankel, under the influence of Riemann, was the founder of the independent theory of functions of a real variable. Now Cantor began his work by studying Hankel's memoir, and he applied his own views on irrationals and derivatives of point aggregates to some of Hankel's theorems. Then Cantor developed the theory of point aggregates as an independent subject, and finally in 1882 defined transfinite numbers independently of the particular aggregates with which they first appeared in mathematics.

But Cantor was most strongly influenced by Weierstrass who first pointed out the logical circle involved in any attempt to define irrationals as limits. Weierstrass gave a theory of real numbers which avoided these errors, but he did not offer a satisfactory theory of the nature of integers. To Cantor we owe the proof that the number continuum cannot be put into one—one correlation with the real algebraical numbers; and that continua of many dimensions can be correlated with those of one. We also owe to him the distinction between cardinal powers and ordinal types; the recognition of orders of infinity among infinite powers; the definition of the ordinal type of the continuum; and the laws of arithmetic for transfinite powers and ordinal types.

Mr. Jourdain has stated and proved a good many of Cantor's earlier theorems in the introduction to this book.

The only criticism to be made on this translation is the large number of misprints that have slipped in. *E.g.* on page 48 C^{ν} is a misprint for C_{ν}; on page 88, line 18, M should be $\overline{\overline{M}}$; on page 76, line 5, we have '$x \geqq$ and $\leqq 1$,' where the meaning appears to be $x \geqq 0$ and $\leqq 1$; on the same page, line 10, Aleph appears for Aleph_0; on page 101, line 6, E appears instead of E^1. There are probably other misprints which I have overlooked.

C. D. BROAD.

The Natural Theology of Evolution. By J. N. SHEARMAN. George Allen & Unwin. 10s. 6d. net.

The author of this book believes that the advance of scientific knowledge has strengthened Paley's famous argument that design in nature proves the existence of an intelligent author of nature. In Paley's own day, we are told, it was never doubted "that his argument was convincing," and if it is doubted in our day it is not on account of any weakness in the reasoning but because it is waiting for some one to carry it a step farther, a step rendered necessary by the modern theory of evolution. To this task the author devotes himself.

At the outset however there appears confusion, or at least the neglect of an important distinction. An argument may be sound or unsound, it may be merely defective and incomplete, or it may conceal an actual fallacy. But even if it be sound it is not necessarily convincing, and an unsound argument may carry conviction. For this a psychological and not merely a logical condition is necessary. It is very doubtful if any one in Paley's day or since has been convinced by the argument, though very many then and since have held it to be sound. If however it is not sound its radical defect lies in its statement and not in any failure to push it as far in Paley's day as it can be in ours. Of such a possible defect the author shows himself nowhere conscious, yet this is surely the one thing needful.

The argument is familiar to every one. A man crossing a heath is supposed to find a watch and after considering it to conclude that it is not a chance shape which matter has assumed but an object contrived and fashioned by an intelligent artificer. What leads him to this conclusion? The recognition of design in the object. But there is the same evidence of design in nature and a notable instance is his own bodily organism. Is he not forced, then, from a consideration of the nature and function of his own body to conclude that it also is the work of an intelligent artificer? But the crux of the argument as it affects us is just whether it is recognition of design that leads the man to conclude the watch is the work of an artificer. Suppose the man crossing the heath is a geologist, that what he finds is a chipped flint, and that what he concludes is the existence of a human being in the remote past. What has led him to the conclusion is not design (he may or may not have been interested in that), it is the recognition of characteristic human work. He has observed that man fashions inert matter into tools for his use, and he recognises the flint as a tool so fashioned.

According to the theory of Bergson, in *Creative Evolution*, it is this power of fashioning inert matter into tools for his use which characterises the intellectual mode of man's activity, whereas man himself is not a tool but a user of tools, not a machine but a centre of free living activity.